CALCULUS BC

Semester 1 Final Part B

Time – 45 minutes

Number of questions – 13

A GRAPHING CALCULATOR IS <u>REQUIRED</u> FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using your scratch paper for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding box on the scantron. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

YOU <u>MAY</u> WRITE ON THIS EXAM.

26. Let f be a continuous function such that $\int_2^3 f(2x)dx = 8$. What is the value of $\int_4^6 f(x)dx$?

(A) 4 (B) 8 (C) 12 (D) 16 (E) 32

Questions 27 and 28 refer to the following information.

The roof and walls of a storage building are built in the shape modeled by the curve $y(x) = 20 - \frac{x^6}{3,200,000}$. Each cross section cut perpendicular to the *x*-axis is a rectangle with a base of 50 feet and a height of y feet.

27. In cubic feet, the volume of the building is approximately

(A) 686 (B) 2,000 (C) 17,100 (D) 34,300 (E) 50,000

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28. What is the average height, in feet, of the storage building described above?

(A) 8.571 (B) 11.920 (C) 12.500 (D) 15.071 (E) 17.143

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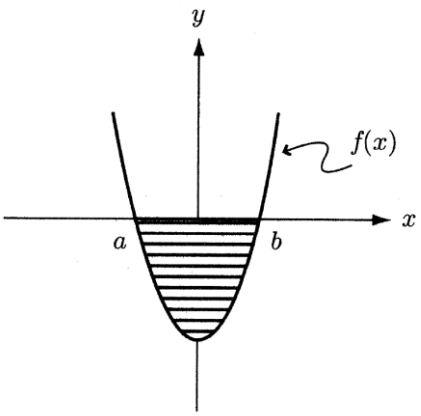
29. A company manufactures x calculators weekly that can be sold for $75 - 0.01x$ dollars each. The cost of manufacturing x calculators is given by $1850 + 28x - x^2 + 0.001x^3$. The number of calculators the company should manufacture weekly in order to maximize its weekly profit is

(A) 611 (B) 652 (C) 683 (D) 749 (E) 754

30.



If f is the continuous function shown in the figure above, then the area of the shaded region is

(A) $\int_a^b f(x)dx$ (B) $\int_b^a f(x)dx$ (C) $\int_b^{-a} f(x)dx$ (D) $\int_{-a}^b f(x)dx$ (E) $\int_{-b}^{-a} f(x)dx$

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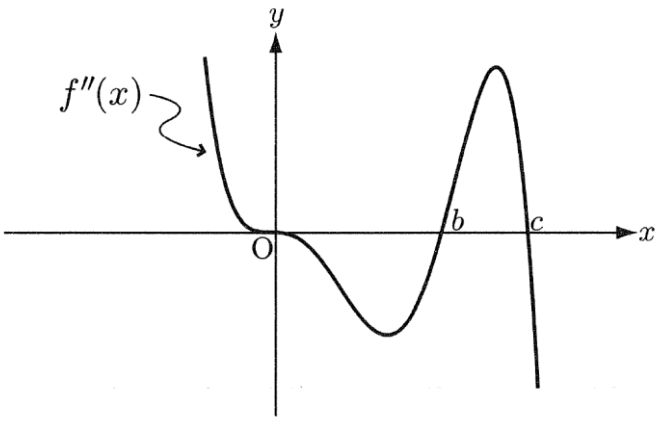
31. A missile rises vertically from a point on the ground 75,000 feet from a radar station. If the missile is rising at the rate of 16,500 feet per minute at the instant when it is 38,000 feet high, what is the rate of change, in radians per minute, of the missile's angle of elevation from the radar station at this instant?

(A) 0.175 (B) 0.219 (C) 0.227 (D) 0.469 (E) 0.507

32.



The figure above shows the graph of $f''(x)$, the second derivative of a function $f(x)$. The function $f(x)$ is continuous for all x. Which of the following statements about f are true?

I. f is concave up for $x<0$ and $b<x<c$.

II. f has a relative minimum in the open interval $b<x<c$.

III. f has points of inflection at $x=0$ and $x=b$.

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

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33. On the interval $[0,b]$, the number $c=4.522$ is guaranteed by the Mean Value Theorem for the function $f(x)=\sin x$. What is the approximate value of b?

(A) 4.000 (B) 5.026 (C) 5.865 (D) 6.259 (E) 12.000

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34. The efficiency of an automobile engine is given by the continuous function $r(c)$ where r is measured in liters/kilometer and c is measured in kilometers. What are the units of $\int_0^5 r(c)\,dc$?

(A) liters (B) kilometers (C) liter-kilometers (D) liters/kilometer (E) kilometers/liter

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35. If $f(x)>0$ is continuous and $g(x)=\int_0^x \sqrt{(f(t))^2-1}\,dt$, what is the length of the graph of $g(x)$ from $x=a$ to $x=b$?

(A) $\int_a^b f(x)\,dx$ (B) $\int_a^b g(x)\,dx$ (C) $\int_a^b \sqrt{(f(x))^2+1}\,dx$

(D) $\int_a^b \sqrt{g(x)+1}\,dx$ (E) $\int_a^b \sqrt{(g(x))^2+1}\,dx$

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36. When $x=\frac{\pi}{4}$, the rate at which $\sin^2 x$ is increasing is k times the rate at which x is increasing. What is the value of k?

(A) $\sqrt{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) 1 (D) $\frac{1}{2}$ (E) -1

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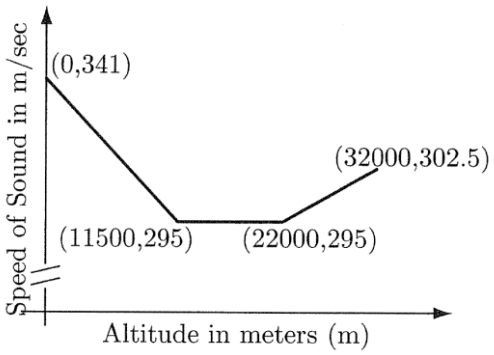
37. If $\int_0^{1000} 8^x\,dx-\int_a^{1000} 8^x\,dx=10.40$, then a is approximately

(A) 1.378 (B) 1.478 (C) 1.578 (D) 1.678 (E) 1.778

38.



In the earth's atmosphere the speed of sound is a function of the altitude. The figure above, consisting of three line segments, shows the speed of sound, $s(a)$, in meters per second as a function of altitude, a, in meters. The graph is not drawn to scale. What is the average speed of sound in meters per second on the interval $[0,32000]$?

(A) 295 (B) 303.9 (C) 304.4 (D) 306.8 (E) 312.8

STOP. THIS IS THE END OF THIS PORTION OF THE FINAL. IF YOU HAVE FINISHED EARLY, GO BACK AND CHECK OVER YOUR WORK. REMAIN SEATED UNTIL INSTRUCTED OTHERWISE.