CALCULUS BC

Semester 1 Final Part A

Time – 55 minutes

Number of questions – 25

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using your scratch paper for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding box on the scantron. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

YOU MAY WRITE ON THIS EXAM.

1. $\int_0^2 xe^x\,dx =$

(A) $1-e^2$ (B) e^2-1 (C) e^2+1 (D) e^4-e^2+1 (E) e^4+e^2-1

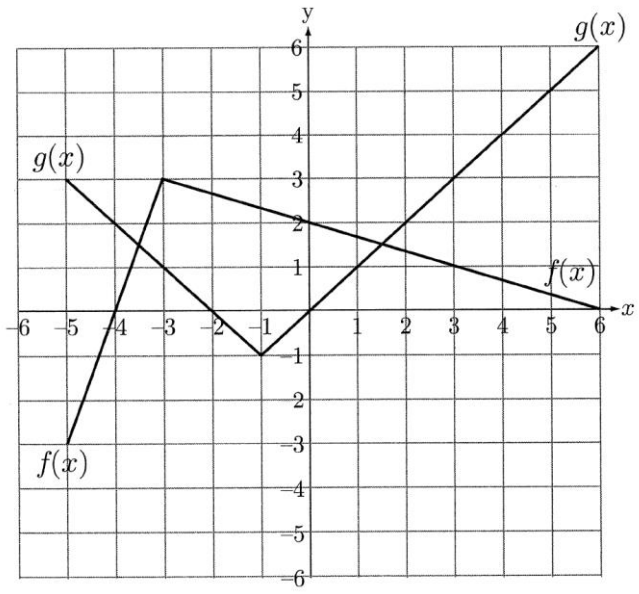
2. An equation of the tangent line to the curve $y=\dfrac{3x+4}{4x-3}$ at the point $(1,7)$ is

(A) $y=-25x+32$ (B) $y=31x-24$ (C) $y=7x$ (D) $y=-5x+12$ (E) $y=25x-18$

3. A particle moves along the *x*-axis so that at any time t its position is given by $x(t)=\dfrac{1}{2}\sin(t)+\cos(2t)$.
What is the acceleration of the particle at $t=\pi/2$?

(A) 0 (B) $\dfrac{1}{2}$ (C) $\dfrac{3}{2}$ (D) $\dfrac{5}{2}$ (E) $\dfrac{7}{2}$

4.



The functions f and g are piecewise linear functions whose graphs are shown above.
If $h(x)=f(g(x))$, then $h'(-3)=$

(A) -3 (B) $-\dfrac{1}{3}$ (C) 0 (D) $\dfrac{1}{3}$ (E) 3

5. $\int_0^3 \dfrac{x}{\sqrt{x^2+16}}\,dx =$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

6. If $y = \ln(3x+5)$, then $\frac{d^2y}{dx^2} =$

(A) $\frac{3}{3x+5}$ (B) $\frac{3}{(3x+5)^2}$ (C) $\frac{9}{(3x+5)^2}$ (D) $\frac{-9}{(3x+5)^2}$ (E) $\frac{-3}{(3x+5)^2}$

7. $\int_{-1}^{1} \frac{dx}{x^2+5x+6} =$

(A) $\ln\frac{3}{2}$ (B) $\ln\frac{1}{4}$ (C) $\ln\frac{2}{3}$ (D) $\ln 6$ (E) $\ln 12$

8. If $y^2 - 2xy = 21$, then $\frac{dy}{dx}$ at the point $(2,-3)$ is

(A) $-\frac{6}{5}$ (B) $-\frac{3}{5}$ (C) $-\frac{2}{5}$ (D) $\frac{3}{8}$ (E) $\frac{3}{5}$

9. The average value of $\sqrt{3x}$ on the closed interval $[0,9]$ is

(A) $\frac{2\sqrt{3}}{3}$ (B) $2\sqrt{3}$ (C) 6 (D) $6\sqrt{3}$ (E) $18\sqrt{3}$

10. If $C(x)$ gives the cost in dollars of producing x items of a certain product, which of the following statements are true about $\frac{dC}{dx}$, the derivative of $C(x)$?

I. The units of $\frac{dC}{dx}$ are dollars per item.

II. The value of $\frac{dC}{dx}$ at any value of x approximates the cost of producing one additional item.

III. $\frac{dC}{dx}$ is the rate at which items are produced.

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

11. On which of the following intervals is the graph of $y = 6x^2 + \frac{x}{2} + 3 + \frac{6}{x}$ concave down?

(A) $x < -1$ (B) $x < 0$ (C) $-1 < x < 0$ (D) $0 < x < 1$ (E) $x > -1$

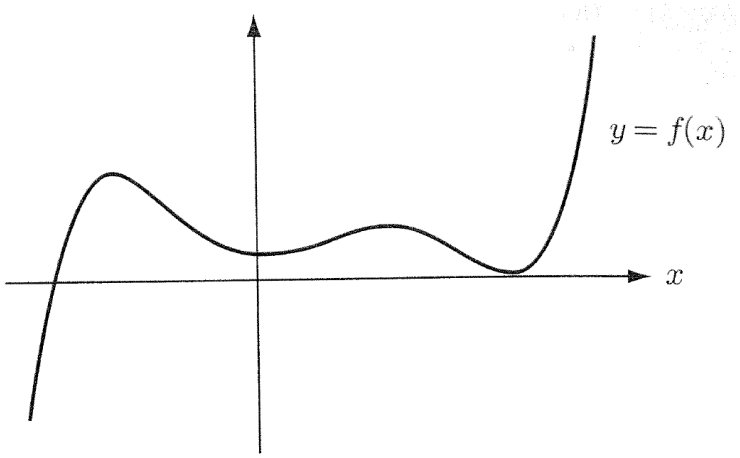
12. What is the area of the largest rectangle with lower base on the *x*-axis and upper vertices on the curve $y = 12 - x^2$?

(A) 8 (B) 12 (C) 16 (D) 32 (E) 48

13. $\frac{d}{dx}\int_0^{2x}\left(e^t + 2t\right)dt =$

(A) $e^{2x} + 4x$ (B) $e^{2x} + 4x - 1$ (C) $e^{2x} + 4x^2 - 1$ (D) $2e^{2x} + 4x$ (E) $2e^{2x} + 8x$

14.



If the graph of a fifth-degree polynomial, $f(x)$, is shown above, then the graph of $f'(x)$, the derivative of $f(x)$, will cross the *x*-axis in exactly how many points?

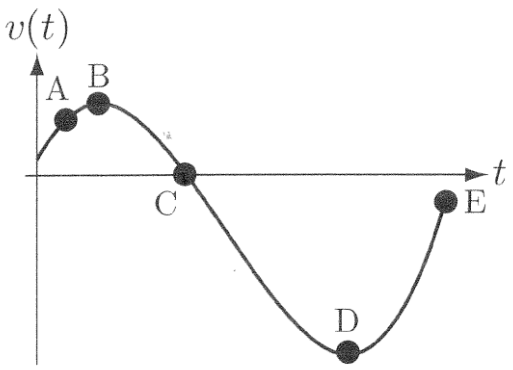
(A) None (B) One (C) Two (D) Three (E) Four

15. At what point on the curve $x^2 - y^2 + x = 2$ is the tangent line vertical?

(A) $(1,0)$ only (B) $(-2,0)$ only (C) $(1,\sqrt{2})$ only (D) $(1,0)$ and $(-2,0)$

(E) The tangent line is never vertical.

16.



The figure above shows the graph of the velocity of an object moving on the *x*-axis as a function of time. Which of the marked points corresponds to the time when the object is farthest to the right?

(A) A (B) B (C) C (D) D (E) E

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17. If $\lim_{x\to 2}\dfrac{f(x)}{x-2} = f'(2) = 0$, which of the following must be true?

I. $f(2) = 0$

II. $f(x)$ is continuous at $x = 2$.

III. $f(x)$ has a horizontal tangent line at $x = 2$.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

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18. If $f(x) = \dfrac{\sin^2 x}{1-\cos x}$, then $f'(x) =$

(A) $\cos x$ (B) $\sin x$ (C) $-\sin x$ (D) $-\cos x$ (E) $2\cos x$

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19. If $\dfrac{dy}{dx} = y\cos x$ and $y = 3$ when $x = 0$, then $y =$

(A) $e^{\sin x}+2$ (B) $e^{\sin x}+3$ (C) $\sin x+3$ (D) $\sin x+3e^{x}$ (E) $3e^{\sin x}$

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20. Let $f(x)$ be a differentiable function with no points of inflection on $[a,b]$. If the definite integral $\int_a^b f(x)dx > T$, where T is the Trapezoidal Rule approximation to $\int_a^b f(x)dx$, which of the following statements about $f(x)$ must be true?

(A) $f(x)$ is linear. (B) $f(x)$ is concave up on $[a,b]$. (C) $f(x)$ is concave down on $[a,b]$.
(D) $f(x)$ is increasing on $[a,b]$. (E) $f(x)$ is decreasing on $[a,b]$.

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21. How many extrema (maxima and minima) does the function $f(x)=(x+2)^3(x-5)^2$ have?

(A) None (B) One (C) Two (D) Three (E) Four

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22. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{3}{n}k\right)^2\left(\frac{3}{n}\right)=$

(A) 0 (B) 1 (C) 4 (D) 39 (E) 125

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23. $\lim_{x\to\pi^-}\frac{\sin x}{\cos x+1}$

(A) 0 (B) 1 (C) ∞ (D) $-\infty$ (E) The limit does not exist

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24. $\lim_{x\to 0}\frac{e^x}{\sin x}$

(A) 0 (B) 1 (C) ∞ (D) $-\infty$ (E) The limit does not exist

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25. $\int_0^2 \frac{1}{4+x^2}dx=$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{8}$ (E) $\frac{\pi}{10}$

STOP. THIS IS THE END OF THIS PORTION OF THE FINAL. IF YOU HAVE FINISHED EARLY, GO BACK AND CHECK OVER YOUR WORK. REMAIN SEATED UNTIL INSTRUCTED OTHERWISE.